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When the payment mode affects the quality of advices. Financial analysts, fund managers, and brokerage commissions.*

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ABSTRACT

"Sell-side" analysts advise fund managers with recommendations to buy or sell a stock. But being compensated with commissions proportional to the amount traded can drive the analyst to bias his advice. In a two-agent model, it is notably shown that the probability of a biased equilibrium to occur increases with commission rate, but decreases with the weight of analyst rating. Moreover, the fund manager can cross-check the recommendation with his own signal –this may represent access to an in-house "buy-side analyst". The fund manager does not necessarily follow the sell-side analyst if its own signal is precise enough . The model hence provides a theoretical rationale for recent empirical results about the independance of sell-side analysts.

JEL Classi…cation : G24, D84

Key Words : Financial Analsysts, Brokerage, Stock Recommendations.

RESUMÉ

Les analystes "sell-side" procurent des recommandations d'achat ou de vente d'actions en Bourse aux gérants de fonds. Ils sont rémunérés proportionnellement aux montants échangés sur les marchés, ce qui peut conduire ces analystes à biaiser leurs recommandations. Dans un modèle à deux agents, on montre que la la probabilité d'occurence d'un équilibre sur-optimiste augmente avec le taux de commission de courtage, mais diminue avec le poids de la notation de l'analyste dans sa rémunération. De plus, le gérant de fonds peut croiser la recommandation avec sa propre information privée -cela fait référence aux analystes "buy-side" interne à la société de gestion de portfeuille. Le gérant ne suit pas nécéssairement l'analyste, si son information privée est su¢ samment précise. Ce modèle corrobore des résultats empiriques récents sur l'indépendance des analystes …nanciers.

Classi…cation JEL : G24, D84

Mots-clefs : Analystes …nanciers, commissions de courtage, recommandations boursières.

Since a handful of years, …nancial market regulators in Europe and the United States are trying to reform the payment mode of …nancial analysts' recommendations. Analysts are suspected to intentionally reveal erroneous informations to the fund managers they are supposed to loyally advise. This is measured by a forecast bias –a gap between the recommendation and the realized stock price– which is generally an optimistic bias: the analyst recommends to buy a stock which price ends up decreasing or stagnating[1].

What is exactly the source of the problem? Several factors are mentioned in the literature. The …rst one is linked to cognitive and psychological bias (for a survey restricted to …nance, see Hilton 2001) : analysts make systematic errors in the decision-making process by using heuristics that leads to fail taking into account all available information, or at least to emphasize or minimize the importance of some information. If such bias are certainly important, they do not su¢ ciently help regulators to implement reforms. The other factors are strategic, "rational" sources of bias. Concerning the relation to the …rms recommended, the analyst is forced to over-optimistic recommendation in order to preserve friendly relationships with the …rm's managers : they are the main information provider to the analyst and could threat to cease relationships if the analyst release unfavorable recommendations (Francis & Philbrick 1993, Lim 2001). The bias can also come from investment banking relationships. Investment bank can pressure the brokerage house and the analyst to issue unduly favorable recommendations (Barber, Lehavy & Trueman 2007, Dugar & Nathan 1995, Hayward & Boeker 1998, O'Brien, Hsiou-Wei & McNichols 2005). Nevertheless, these aspects have already received regulatory answers which begin to be detected in available databases (Kadan, Madureira, Wang & Zach 2006).

Another source of bias has been paid less attention : the incentive to generate brokerage commissions. A "sell-side" analyst is a member of a brokerage house. Its pro…t comes from a percentage of trade orders executed in the stock market. Their main clients are fund managers. Besides the execution of orders, the brokerage house can o¤er …nancial services including private access to analysts' recommendations. The payment of such services are bundled with the commissions[2]. This payment system can entail a con‡ict of interest if the expected revenue from a recommendation depends on the volume of trade

[1] Analyst issue several other forecasts, such as earnings forecasts. Considering forecasts or recommendations does not dramatically change the biased nature of the advice. In the model we will focus on recommendations.

[2] Recent reforms has introduced the possiblity to unbundle the payments. In United Kingdom and France, a fund manager can assign a part of commissions to a …rm which is di¤erent than the …rm which execute the order. But this does not change the fact that the payement of the recommendation is proportional to the volume of trade.

that this recommendation will incur. If the analyst thinks that a loyal recommendation is not the most likely to generate trade commissions, he then faces a dilemma between being reputed as a loyal analyst, and generating trading commissions for his …rm.

Can one illustrate the extent, and the limits, of the importance of brokerage commissions in the advent of exaggeratedly optimistic recommendations? It is what this study tries, using a two-agents model, in which an analyst exchanges with a fund manager a recommendation against some volume of trade. It is shown that the rate of commission is positively related to the probability of occurrence of a biased equilibrium, i.e. the situation where the analyst recommends the purchase of a stock although he thinks that the fall is more probable, and that the best response of the fund manager, taking into account his revised beliefs and available information, is to follow the opinion of the analyst. Nevertheless, this is particularly tempered by (1) the information precision that the fund manager privately receives, which illustrates the importance of cross-checking information, for example by paying an in-house "buy-side" analyst; and (2) the possibility to ex-post rate the analyst, i.e. by comparing his recommendation to the realized state of the world. Furthermore, the model allows the fund manager to trade against the recommendation of the analyst. This can explain the empirical results comparing buy- and sell-side analysis presented in (Cheng, Liu & Qian 2006)

The general architecture of the model is that of the reputational herding models, in which an adviser delivers a forecast to a receiver who then acts on the basis of this forecast. One can refer in particular to studies which applied this model to …nancial analysts (Trueman 1994, Graham 1999)[3]. These models are themselves related to seminal works on signalling games.

The description of behaviors (section 1) follows the logic of the model of Rachel Hayes (1998). It is, as far as I know, the …rst formalization of the incentive to generate trading commissions. However the model does not directly deal with strategic choice by the analyst between honest or biased recommendation, but with the decision to cover or not a company, and with the degree of e¤ort (and thus of quality) included in the production of information. The result is that the brokerage incentive leads to favor stocks for which one anticipates a high volume of trade.

[3]Brett Trueman (1994) tackles the idea that the forecasts of the …nancial analysts re‡ect their private information exactly. It shows that is not inevitably the case since analysts seek to be favorably judged by the investors, in order to build a reputation. In this case, one can reveal forecasts which are too close of those put forward before by the other analysts, compared to what would require its private information, . The model of John Graham (1999) shows that analysts can discard their private information, this with an aim of appearing precise, skilful, and of building a good reputation. It speci…es moreover that the propensity to put forth a biased recommendation varies according to certain factors, for example, it decreases with the precision of signals received by the analyst.

The de…nition of equilibrium in section 2 owes much to the model of Andrew Jackson (2005). This model draws on the previous one, adding the Bayesian statistics of reputation games. The choice is no longer about a continuous e¤ort function (the e¤ort e 2 $[0, 1]$), but about a binary choice (favorable / unfavorable recommendation). The author shows the existence of a biased equilibrium in which the analyst chooses a favorable recommendation in order to generate trading volume, which overrides the objective of reputation reached with a loyal recommendation. He shows in particular that biased equilibrium is all the more probable since the investor is naive (he confers to the analyst a high probability of being loyal), but all the less probable when the analyst has precise information and a good reputation. We will refer to the Hayes-Jackson model to indicate this common structure.

The new propositions of the model presented here, and some Hayes-Jackson propositions with more parsimonious assumptions, are mentioned in section 3. The uncertain "type" of the analyst is only about his incentive to bias the recommendation, but there is no need to consider a supplementary uncertain "type" about his signal's precision ("smart"/"dumb" analysts). The case of "naive" investors is not considered, that is why we refer to "fund manager". The fund manager has a private signal at his disposal, which allows to endogenously deviate from the analyst. There is no need to assume an exogenous probability of short sale constraint, as in the Hayes-Jackson model.

Section 4 brie‡y discusses regulatory reforms of sell-side analysts' payment mode, in the light of the results.

1 Environment and behaviors

1.1 Environment

Suppose a fund manager who has access to the information delivered by a sell-side analyst, in exchange from what the manager trades with the analyst's brokerage …rm. Information refers to only one risky asset, a stock, which is perfectly divisible and whose price is P_0 at the initial period t_0. There are two possible states of the world in the future, the favorable state H, in which the price of the risky is x_H, and the unfavorable state L, in which the price is x_L. The state of the world is realized but in t_0 but unobservable, it will be revealed only in t_2. We will use by convenience the term "future state" as a shortcut to indicate "price observed after revelation of the favorable or unfavorable state", x_H and x_L. Prices are such that :

$$x_L < P_0 < x_H \tag{1}$$

At period t_0, every agent observes the same probabilities λ :

$$\begin{aligned} \Pr(x_H) &= \lambda = 0.5 \\ \Pr(x_L) &= 1 - \lambda = 0.5 \end{aligned} \tag{2}$$

So that λ is the initial belief of agents.

At t_0, each agent privately and receive a signal about the future state of the world. The analyst receive s_H (favorable) or s_L (unfavorable), the fund manager receive y_H (favorable) or y_L (unfavorable). This signal y can be viewed as internal information transmitted by a buy-side analyst. As signals result from a personal effort of possibly differing data interpretation, or from the private acquisition of new information, signals are independent[4].

They all know the accuracy of their signals, which are:

$$\Pr(s_H \mid x_H) = \Pr(s_L \mid x_L) = p \tag{3a}$$

$$\Pr(y_H \mid x_H) = \Pr(y_L \mid x_L) = z \tag{3b}$$

Let us define parameters p and z as the probabilities of receiving "correct" signals.

It is assumed that the analyst receives a more precise signal than the fund manager. This is meant to capture the fact that the analyst is limited to a small number of stocks, contrary to the fund manager who must screen a huge set of investment opportunity, as the risky asset is only one element of his wealth[5].

Assumption 1 $\frac{1}{2} < z < p \leq 1$

Note that following the fund manager's signal is always strictly better than a random choice $(1/2)$, but strictly worse than a "perfect" signal (1). The probabilities to receive "incorrect" signals are:

$$\Pr(s_H \mid x_L) = \Pr(s_L \mid x_H) = 1 - p \tag{4}$$

$$\Pr(y_H \mid x_H) = \Pr(y_L \mid x_L) = 1 - z \tag{5}$$

The fund manager must decide the amount of risky asset to possess. He has to revise his initial belief λ with the help of his signal y. He cannot observe the signal of the analyst s but only his recommendation r (with r_H the favorable and r_L the unfavorable recommendation). Thus he will decide depending on r and y.

[4] Suppose for example that the analyst uses a bottom-up approach and the fund manager a top-down approach.

[5] Saying this means that I have in mind "stock-picking" style fund managers. Index fund only replicates a stock index, thus they do not need to pay for fundamental analysis.

The analyst must decide which recommendation r to reveal. He has two objectives : to generate trading commissions, and to enhance his reputation. The reputation is given by the fund manager, when he rates the analyst, by comparing the observed state of the world with the recommendation the analyst gave him.

The aim of the model is to study a Nash equilibrium. At time t_1, the analysts sends r, which is a best response to the fund manager's strategy, and the fund manager define a traded volume V, which is a best response to the strategy of the analyst. At t_2, the state H or L is revealed, and the fund manger rates the analyst. This kind of game need not be repeated. Like in (Ramakrishnan & Thakor 1984, Scharfstein & Stein 1990, Graham 1999), the fact to assess ex-ante the expected ex-post reputation is sufficient to ensure that it is not a dominant strategy for the analyst to systematically lie. It is not necessary to build a reputation through time, as in Sobel (1985).

Before going farther, let us precise that when variables are indexed by the binary states H and L, the index is not mentioned when all cases are considered. For example, mentioning (s, r) refers to (s_H, r_H), (s_H, r_L), (s_L, r_H), (s_L, r_L). Mentioning the couple $(s = r)$ refers to the subcases where indexes are the same, i.e. (s_H, r_H) and (s_L, r_L). Hence $(s \neq r)$ refers to the subcases where indexes are different, i.e. (s_H, r_L) and (s_L, r_H).

1.2 Fund Manager's behavior

1.2.1 Trading volume decision

As in Hirshleifer (1971) and Jackson (2005), the utility function is logarithmic, $U = \ln(W + \text{net gain})$ where W is the the wealth of the fund manager apart from the risky asset. With P_0 the price of the stock at t_0 and γ_0 the quantity owned at t_0, if the agent invested γ_0 stocks and that the price becomes x_H, his net gain is $\gamma_0(x_H - P_0)$. If the price becomes x_L, the net gain is $\gamma_0(x_L - P_0)$. At t_0 the fund manager must chose the optimal level of quantity to hold, γ_0^*.

Thus he must maximize expected utility:

$$E(U) = \lambda \ln(W + \gamma_0(x_H - P_0)) + (1 - \lambda) \ln(W + \gamma_0(x_L - P_0)) \tag{6}$$

with respect to γ, which yields γ_0^* :

$$\gamma_0^* = \frac{-W[\lambda(x_H - P_0) + (1 - \lambda)(x_L - P_0)]}{(x_H - P_0)(x_L - P_0)} \tag{7}$$

Now consider as the fund manager's initial endowment. At t_1, he has to modify this quantity, as long as he revises his initial belief conditionally to his own signal and the recommendation. The revised belief λ' is:

$$\begin{aligned} \lambda' &= \Pr(x_H \mid y, r) \\ 1 - \lambda' &= \Pr(x_L \mid y, r) \end{aligned} \tag{8}$$

By analogy he can compute γ_1^*, which gives the same result as (7) but with λ' instead λ. Computing the difference $\gamma_1^* - \gamma_0^*$ gives the volume traded.

$$\begin{aligned} \gamma_1^* - \gamma_0^* &= (\lambda' - \lambda) * \frac{W[(x_L - P_0) + (x_H - P_0)]}{(x_L - P_0)(x_H - P_0)} \\ V &\equiv (\lambda' - \lambda) * \alpha \end{aligned} \tag{9}$$

Where α is a strictly positive amount and V the volume traded on the basis of available information.

1.2.2 Analyst rating decision

As in Ramakrishnan & Thakor (1984) and Jackson (2005) the analyst is evaluated ex-post. But I use this device here in order not to postulate the natural and intangible division between "smart" and "dumb" analysts. The manager does not have as a concern to guess if the analyst is smart or dumb, but to know if its recommendation will be right. Formally that does not dramatically changes things, but the interpretation of the model will be thus in conformity with the observation. Indeed, some empirical investigations showed that fund managers pays less attention to analyst ranking than to the comparison between his own information and the one given by the sell-side analyst[6].

Let the rating of the analyst vary between 0 and 1. At the end of the game, the fund managers compares the recommendation r with the price x, and decides to increase the rating by n if it was correct ($r = x$) or to decrease it by the same amount n if it appears to be incorrect ($r \neq x$).

To simplify, assume that the initial rating is $N_0 = 0.5$. The the range of n is $n \in [0, 1/2]$ and it gives the final rating $N \in [0, 1]$, which is defined by:

$$\begin{cases} N = 0.5 + n \text{ if } r = x \\ N = 0.5 - n \text{ if } r \neq x \end{cases} \tag{10}$$

The scale of N bears no importance, we will see that only the variation of the rating (n) has a role to play.

[6] See for example Barker (1998) or Galanti (2006).

1.3 Analyst's behavior

The first objective of the analyst is to generate trading commissions. Let $c \in [0, 1]$ be the percentage trading commission rate. The analyst must compute $E(V)$,the expected trading volume. It depends on the belief of the fund manager about the future price, and this belief in turn depends on the recommendation $r,$ and on the fund manager's signal y. As long as the analysts ignores y, the best he can do is to use his own signal as a proxy. Hence he will compute $cE(V \mid r, s)$.

The second objective is to have the best possible rating. Let $k \in [0, \infty)$ be the weight of the rating in the analyst's utility. A high (low) k means that the analyst do (not) care a lot about reputation. The rating depends on the comparison between r and x, unknown at the date of computation. Again he will use x, the best available signal about $x.$ The program of the analyst is then:

$$\max_r \{c.E(V \mid r, s) + k.E(N \mid r, s)\} \tag{11}$$

As the choice is binary, the obvious maximization consists in comparing the two possible cases. The analyst will choose r_L if:

$$c.E(V \mid r_L, s) + k.E(N \mid r_L, s) > c.E(V \mid r_H, s) + k.E(N \mid r_H, s) \tag{12}$$

and r_H in the contrary.

Our goal is to illustrate which parameters determine an equilibrium with a biased recommendation. The term biased recommendation means that it is different from the signal received ($s \neq r$) , and loyal recommendation means that it is accorded to the signal ($s = r$).

It is then necessary to assume that there exists some asymmetry in the model to generate non-trivial results. In Sobel (1985), Sharstein and Stein (1990) and Graham (1999) and Jackson (2005) for instance, they posit a probability of being a smart analyst (with precise signals) or a dumb one (with non-informative signals). I posit that the asymmetry depends on the nature of the message sent:

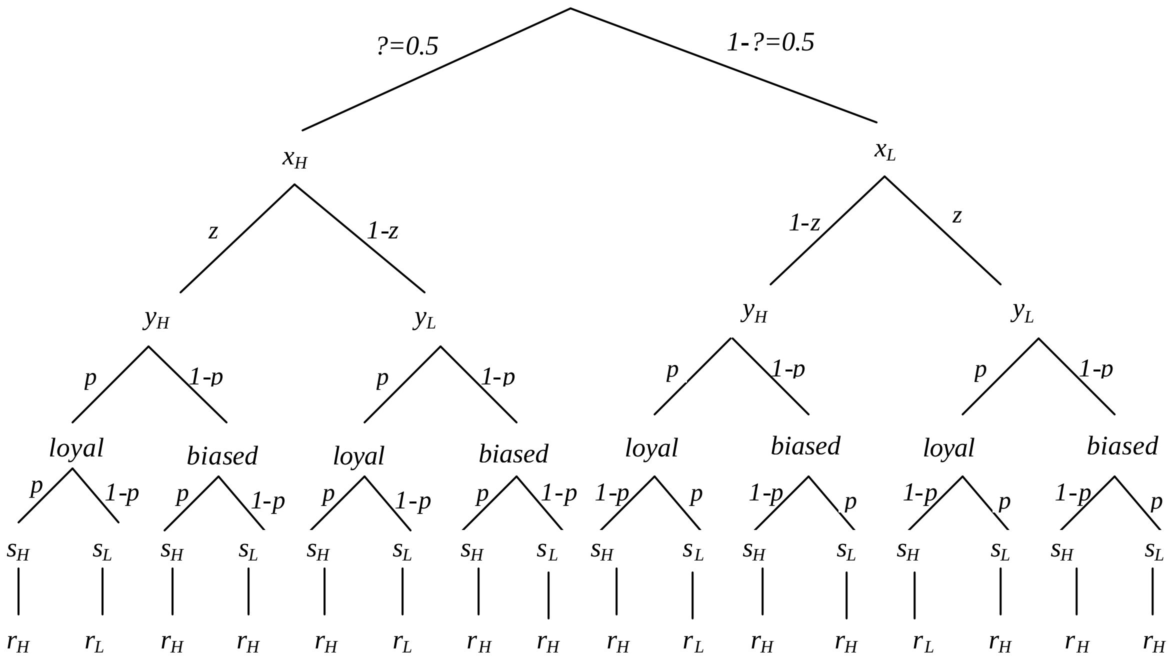
Assumption 2 If the analyst receives a favorable signal, he sends a loyal recommendation (if s_H, then r_H). If the analyst receives an unfavorable signal, then with probability π he sends a loyal recommendation (if s_L, then r_L), and with probability $1 - \pi$, he sends a biased recommendation (if s_L, then r_H)

In the following we will define as a biased (loyal) equilibrium the one containing biased (loyal) recommendation. It is important to insist that the previous assumption is not an ad hoc assumption, as long as 1/ the goal is not to generate a biased equilibrium but to study the parameters that cause its occurrence, 2/ the analyst must still compute (12) to make his decision, 3/ the bias is not automatic, but only probable, 4/ the probability π can be fully de…ned and depends on the parameters of the model, as we will see in the next section, and 5/ even in a biased equilibrium, the fund manager can trade in the opposite direction than the one indicated by the (possibly biased) recommendation.

1.4 Information structure

Figure (1) subsume the previous paragraphs by representing the information structure.

Figure 1: Information structure



Variables "on" the branches are probabilities, those at the nodes are the corresponding events. The signals (y, s) have probabilities (respectively z and p) conditioned to the state realized (x). Probability π is independent from the realization of the state and from signal y. The recommendation r depends on the event loyal / biased and on the signal s. The concept solution used here to determine the equilibrium is the prefect Bayesian equilibrium, which is de…ned as sequentially rational given the beliefs of the agents.

2 Equilibrium characterization

The perfect Bayesian equilibrium is the solution concept used to identify an equilibrium in the game. The agents decisions are sequentially rational given their beliefs, and the beliefs are revised using Bayes' rules. We have seen that the volume traded V is the best response for the fund manager, and that playing r_L given s_H is an off-the-equilibrium path. Thus, to define equilibrium, we must study the choice of the analyst. One can rewrite his decision rule (12) as follows. The analyst will choose r_L if:

$$k.\{E(N \mid r_L, s) - E(N \mid r_H, s)\} > c\{E(V \mid r_H, s) - E(V \mid r_L, s)\} \tag{13}$$

and r_H in the contrary. The sign of the differences between expected ratings, and expected volumes, depend on the signal received by the analyst. Let us study first the expected rating difference, then the expected volume.

2.1 Expected rating

Consider s_L, then s_H.

2.1.1 Expected rating with unfavorable signal s_L

The aim is to compute:

$$E(N \mid r_L, s_L) - E(N \mid r_H, s_L) \tag{14}$$

Following (10) if s_L and r_L are realized, the rating becomes 0.5 + n if x_L is observed or 0.5 $-$ n if x_H is observed. Note that x is conditional on s but not on r, since it is the knowledge of s that gives information to the analyst and obviously not the decision about r, a redundant and potentially biased information r.Expecting the rating considering the unfavorable recommendation case (r_L) gives:

$$\begin{aligned} E(N \mid r_L, s_L) &= (0.5 - n)\Pr(x_H \mid s_L) + (0.5 + n)\Pr(x_L \mid s_L) \\ &= (0.5 - n)(1 - p) + (0.5 + n)p \\ &= 0.5 + n(2p - 1) \end{aligned}$$

and in the favorable recommendation (r_H) case:

$$\begin{aligned} E(N \mid r_H, s_L) &= (0.5 + n)\Pr(x_H \mid s_L) + (0.5 - n)\Pr(x_L \mid s_L) \\ &= (0.5 + n)(1 - p) + (0.5 - n)p \\ &= 0.5 - n(2p - 1) \end{aligned}$$

Note that the strength of the rating revision, expressed by $n(2p - 1)$, depends positively on the analyst's signal precision p. Rewriting (14) it is straightforward to show that it is always positive since $\Pr(x_H) = \lambda = 0.5$:

$$E(N \mid r_L, s_L) - E(N \mid r_H, s_L) = 2n(2p - 1) > 0 \tag{15}$$

that means that conditional to s_L, on average, the rating of a loyal recommendation is always higher than the rating of a biased recommendation.

2.1.2 Expected rating with favorable signal s_H

Again, we compute:

$$E(N \mid r_L, s_H) - E(N \mid r_H, s_H) \tag{16}$$

which gives:

$$= -2n(2p - 1) < 0 \tag{17}$$

This means that on average and whatever the signal received, the rating of a loyal recommendation is always higher than the rating of a biased recommendation. Although unsurprising, this reputation result is necessary for the consistency of the model. It is obtained in a different manner from the Hayes-Jackson model. The fund manager cannot observe the intentions of the analyst (his loyalty) by comparing his signal s to his recommendation r, but by rating the analyst on the basis of his acts, he has an efficient credibility constraint device at his disposal.

2.2 Expected trade

After having examined the left member of expression (13) we now turn to the right member.

2.2.1 Expected trade with unfavorable signal s_L

We study the sign of:

$$E(V \mid r_H, s_L) - E(V \mid r_L, s_L) \tag{18}$$

As seen in (9), the traded volume V depends on the revised belief of the fund manager, denoted $\lambda'(y, r) = \Pr(x \mid y, r)$. Details about $\lambda'(y, r)$ are given in annex A (p.25).

We have:

$$
\begin{aligned}
E(V \mid r_H, s_L) &= \alpha(\lambda^0(y_H, r_H) - 0.5)\Pr(y_H, r_H) \\
&\quad + \alpha(\lambda^0(y_L, r_H) - 0.5)\Pr(y_L, r_H) \\
&= \alpha\{2z(1-(1-p)\pi) - \frac{1}{2}\}(\frac{1}{2} - \frac{1}{4}\pi) + \alpha\{2(1-z)[1-(1-p)\pi] - \frac{1}{2}\}(\frac{1}{2} - \frac{1}{4}\pi) \\
&= \alpha(\frac{1}{2} - \frac{1}{4}\pi)[2(1-(1-p)\pi) - 1] \\
&= \alpha(\frac{1}{2} - \frac{1}{4}\pi)(1 - 2(1-p)\pi)
\end{aligned} \tag{19}
$$

Idem:

$$
\begin{aligned}
E(V \mid r_L, s_L) &= \alpha(\lambda^0(y_H, r_L) - 0.5)\Pr(y_H, r_L) \\
&\quad + \alpha(\lambda^0(y_L, r_L) - 0.5)\Pr(y_L, r_L) \\
&= \alpha\{2z(1-p)\pi - \frac{1}{2}\}\frac{1}{4}\pi + \alpha\{2(1-z)(1-p)\pi - \frac{1}{2}\}\frac{1}{4}\pi \\
&= \alpha\frac{1}{4}\pi\{2(1-p)\pi - 1\}
\end{aligned} \tag{20}
$$

We can now study the sign of :

$$
\begin{aligned}
& E(V \mid r_H, s_L) - E(V \mid r_L, s_L) \\
&= \alpha(\frac{1}{2} - \frac{1}{4}\pi)(1 - 2(1-p)\pi) - \alpha\frac{1}{4}\pi\{2(1-p)\pi - 1\} \\
&= \alpha\frac{1}{2}(1 - 2(1-p)\pi)
\end{aligned}
$$

Since α is positive, and $\pi \in [0,1]$ and $0 < (1-p) < 0.5$ (because p is strictly superior to $\frac{1}{2}$ since there always is a signal of precision z such that $0.5 < z < p$); we have: $-2(1-p)\pi \in [-1, 0]$ and $1 < (1 - 2(1-p)\pi) < 2$. The sign of the difference is then strictly positive (which joins up with proposition 1, iv, c of Jackson 2005).

Proposition 1 When the analyst receives an unfavorable signal, the expected trading volume conditional to a biased recommendation is always superior to the expected trading volume conditional to a loyal recommendation: $E(V \mid r_H, s_L) > E(V \mid r_L, s_L)$

2.2.2 Expected trade with favorable signal

We study the sign of:

$$E(V \mid r_H, s_H) - E(V \mid r_L, s_H)$$

Note that the case s_H, r_L is excluded by assumption 2. First we have:

$$E(V \mid r_H, s_H) = \alpha(\lambda'(y_H, r_H) - 0.5)\Pr(y_H, r_H) + \alpha(\lambda'(y_L, r_H) - 0.5)\Pr(y_L, r_H)$$

We can remark that this is by definition equal to $E(V \mid r_H, s_L)$, hence:

$$E(V \mid r_H, s_H) = \alpha(\frac{1}{2} - \frac{1}{4}\pi)(1 - 2(1 - p)\pi)$$

and:

$$E(V \mid r_L, s_H) = \alpha(\lambda'(y_H, r_L) - 0.5)\Pr(y_H, r_L) + \alpha(\lambda'(y_L, r_L) - 0.5)\Pr(y_L, r_L)$$

which is by definition equal to $E(V \mid r_L, s_L)$:

$$E(V \mid r_L, s_H) = \alpha\frac{1}{4}\pi\{2(1 - p)\pi - 1\}$$

Thus we can express:

$$\begin{aligned} E(V \quad &\mid \quad r_H, s_H) - E(V \mid r_L, s_H) \\ &= \quad \alpha\frac{1}{2}(1 - 2(1 - p)\pi) > 0 \end{aligned}$$

Then we can complete proposition 1,

Proposition 2 The expected volume of a favorable recommendation is always superior to the expected volume of an unfavorable recommendation, may the recommendation be biased, hence $E(V \mid r_H, s_H) > E(V \mid r_L, s_H)$; or loyal (prop.1), hence: $E(V \mid r_H, s_L) > E(V \mid r_L, s_L)$

We can now state that the behavior that is forbidden by assumption 2 (playing r_L following s_H) is not an equilibrium behavior, because the analyst would be losing both from the reputation point of view –as biased recommendation entails a lower rating, see (17)– and from the trading volume commission point of view –it is proposition 2.

2.3 Decision rule

We can now comment the analyst decision rule (13). The analyst will choose r_L if:

$$k.\{\underbrace{E(N \mid r_L, s) - E(N \mid r_H, s)}_{\substack{>0 \text{ if } s_L \text{ (loyal)} \\ <0 \text{ if } s_H \text{ (biased)}}}\} > c\{\underbrace{E(V \mid r_H, s) - E(V \mid r_L, s)}_{\text{always } >0}\}$$

Thus if s_H is received, the inequality above is never true, then the analyst will choose r_H. If s_L is received, then the result depends on the weighting parameters c and k.[7] Isolating $k,$ the analyst will choose r_L if:

$$k \geq \frac{c\{E(V \mid r_H, s) - E(V \mid r_L, s)\}}{\{E(N \mid r_L, s) - E(N \mid r_H, s)\}}$$

Let us define k^*, the $k-$threshold which determines which recommendation to choose when receiving s_L :

$$k^* = \frac{c\{E(V \mid r_H, s_L) - E(V \mid r_L, s_L)\}}{\{E(N \mid r_L, s_L) - E(N \mid r_H, s_L)\}} \tag{21}$$

So that the decision rule of the analyst finally is :

$$\begin{cases} \text{if } s_H \text{ then } r_H \\ \quad \text{if } s_L \text{ then} \\ r_L \text{ if } k \geq k^* \\ r_H \text{ if } k < k^* \end{cases} \tag{22}$$

Now the equilibrium situation can be defined.

Definition 1 The recommendation r of the analyst which complies with (22), and the corresponding trading volume $V = \alpha(\lambda^0(y, r) - 0.5)$ chosen by the fund manager, are forming an equilibrium.

The parameter k, which represents the weighting of rating in the analysts payoff, i.e. the importance of reputation, help us determine the probability π mentioned in assumption 2:

$$\begin{aligned} \Pr(k \geq k^*) &= \pi \\ \Pr(k < k^*) &= 1 - \pi \end{aligned} \tag{23}$$

This confirms π as being the probability that the analyst is loyal and $1 - \pi$ the probability that he is (optimistically) biased. Hence this must be seen, respectively, as the probabilities of a loyal (biased) equilibrium to occur. In the next section we specify how k^* reacts to shocks on the other parameters and study if the fund manager can "de-bias" the recommendation.

[7] It is assumed than if the two side of the equation are equal, the analysts chooses to be loyal, so we now write as a "suprior or equal" inquality.

3 Equilibrium properties

The aim of this section is to examine how k^*, and hence, how the probabilities of occurrence of equilibria can occur. This will help us draw some conclusion about the bias in analysts recommendations.

3.1 Study of threshold k^*

Rewrite the previous definition (21) as:

$$k^* = \frac{\frac{c\alpha}{2}(1 - 2(1 - p)\pi))}{2n(2p - 1)} \tag{24}$$

This expression shows that k^* depends on parameters c, p, n and α (with α being calculated from the wealth W and prices P_0, x_H and x_L). But it also depends on π. As by definition, π is a function of k^* since $\Pr(k \geq k^*) = \pi$, we must explicitly posit a cumulative distribution function for $f(k)$. For the sake of simplicity, I have chosen the continuous uniform distribution.

3.1.1 The $f(k)$ function

The choice of probability distribution is not important to our subject. However, the two boundaries a and b of a uniform distribution must be chosen in order to ease the interpretation of the results. So we must restrict the support of k, which was initially any real positive number. After some simulations, it appeared that 0 and 100 were satisfying boundaries as they admit the two possible behavior from the analyst (loyal or biased) for large set of parameters. Now define:

$$f(k) = \Pr(k < k^*) = 1 - \pi$$

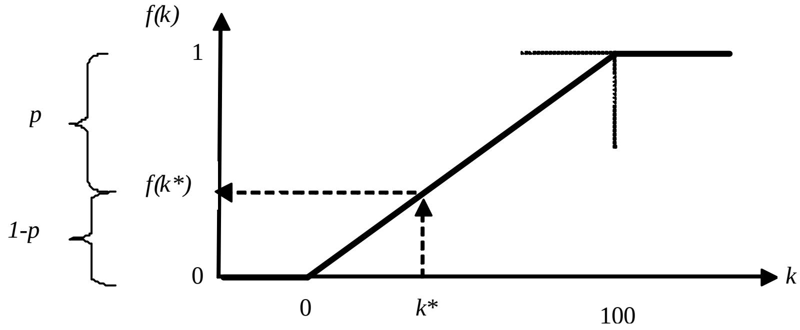
with the properties of a probability distribution following a continuous uniform cumulative density function with parameters $(0, 100)$:

$$\begin{aligned} f(k) &= 0 \text{ if } k \leq 0 \\ f(k) &= \frac{k}{100} \text{ if } 0 < k < 100 \\ f(k) &= 1 \text{ if k} \geq 100 \end{aligned}$$

It is as if the analyst would sort randomly a real number between 0 and 100, each time he has to send a recommendation. This number k gives him the weight of rating (i.e. of reputation) in his utility function.

The function is given in figure (2).

Figure 2: Cumulative density function



Let us take a numerical example. Suppose that $k^* = 25$. Then $f(k^*) = \frac{25}{100} = \Pr(k < k^*) = 1 - \pi$. Hence $\pi = 0.75$. In general terms, we can define:

$$\pi = 1 - \frac{k^*}{100} \tag{25}$$

The next two subsections interpret this expression and explain the sequence of events.

3.1.2 Computation of k^*

From the original expression (24), we simply replace π with $1 - k^*/100$. After some calculations –details in annex B, (p.27)– it yields:

$$k^* = \frac{c\alpha(1 - 2(1 - p))}{4n(2p - 1) - \frac{2}{10^2}c\alpha(1 - p)} \tag{26}$$

This is the expression of k^* we will use henceforth. Let us take a numerical example in order to illustrate what this threshold means. Suppose that the following parameters holds:

α	c	n	p
300	0.03	0.05	0.8

Replacing with (26) we round off $k^* = 64,2$. It means that $\pi = 1 - \frac{64.2}{100} = 35.8\%$. The probability for the analyst to be loyal is 35.8%, the probability of the analyst to be biased is 64.2%. If we now replace π by its value 0.358 in the original expression of k^* (i.e. 24), that confirms $k^* = 64.2$.

Studying how k^* reacts to shocks on parameters α, c, n and p is useful, because:

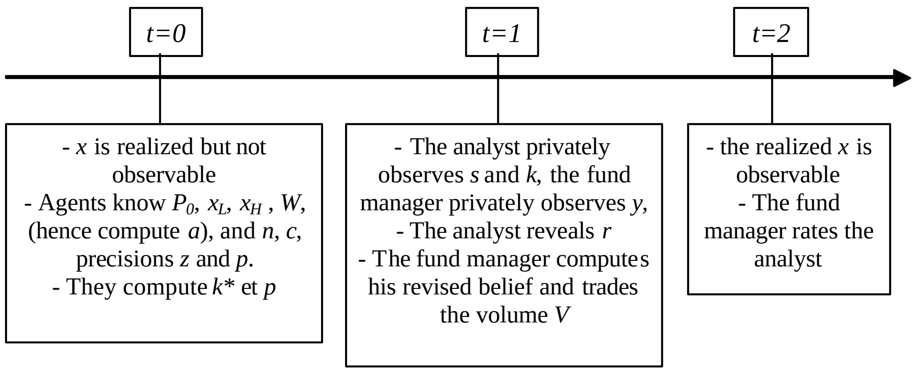
- if k^* increases, the probability of the biased equilibrium to occur (i.e. an equilibrium with a favorable analyst recommendation and a unfavorable analyst signal) increases. Because of the equilibrium definition (see 22 and sq.) and of the uniform distribution chosen for π, then, all else equal, the higher k^* is, the more there are chances that the number k randomly sorted by the analyst is under this threshold k^*, hence the more there are chances that the analyst chooses r_H after receiving s_L , at equilibrium.
- Similarly, if k^* decreases, the probability of the biased equilibrium to occur decreases. Because, all else equal, the lower k^* is, the more there are chances that the number k randomly sorted by the analyst is above this threshold k^*, hence the less there are chances that the analyst chooses r_H after receiving s_L , at equilibrium.

Note that writing "the probability to be loyal (biased)" is then similar to "the probability that the equilibrium with loyal (biased) recommendation occurs".

3.1.3 The sequence of events

The timeline is given in figure (3).

Figure 3: Timeline



Now take a numerical example where a biased equilibrium arises. The parameter values are the following.

α	c	n	p	z
20	0.1	0.1	0.8	0.55

- At $t = 0$, agents know the fund manager wealth $W = 40$, the stock price $P_0 = 4$, the price that would prevail in the favorable state of the world $x_H = 10$, and in the unfavorable state $x_L = 1$. From (9) they compute $\alpha = 20$. They know that, $n = 0.1$, meaning that the rating of the analyst initially equal to 0.5 will be modified by a step of 0.1. The rating will increase (decrease) up to 0.6 (down to 0.4) if the recommendation do (do not) correspond to the price x that will be observed in $t = 2$. Agents also know that the percentage trading commission rate is $c = 0.10$, the fund manager's signal precision is $z = 0.55$ and the analysts's signal precision is $p = 0.80$, which allows them to compute k^* and π. With (26) they compute $k^* = 5.1724$ and with (25) they obtain $\pi = 94.83\%$. The fund manager estimates that the analysts has 94.83% chances of being loyal, and the analyst takes this into account in his computation of the expected trade by the fund manager.

- At time $t = 1$, the fund manager privately observes $y = y_L$, and the analyst privately observes $s = s_L$ and $k = 5$. Since $k < k^*$, the analyst knows his interest is to reveal a biased recommendation r_H. He can check it by comparing the two expected utilities from (12)

$$
\begin{aligned}
E(U(r_L)) &= c.E(V \mid r_L, s_L) + k.E(N \mid r_L, s_L) = 3.094 \\
E(U(r_H)) &= c.E(V \mid r_H, s_L) + k.E(N \mid r_H, s_L) = 3.832
\end{aligned}
$$

So the analyst reveals r_H. The fund manager's two sources of information are hence conflicting. To decide, he computes his revised belief:

$$\lambda'(r_H, y_L) = 0.729$$

The result shows that the price x_H is now more probable than it was ($\lambda = 0.5$). Certainly because of the relatively high precision of the analyst and of the high probability of being loyal, the best response of the fund manager is to find the analyst credible and buy the stock, discarding his own unfavorable signal y_L. The equilibrium trading volume he chooses is:

$$V^* \equiv (\lambda' - \lambda) \times \alpha = 0.229 \times 20 = 4.586$$

- At $t = 2$, the price x_L is revealed. The fund manager ends up with a net loss of $(x_L - P_0) = -13,758$. If he had received a loyal recommendation, the net gain would have been $V(x_H - P_0) = 74,516$. The fund manager downgrades the rating of the analyst, which becomes 0,4.

Note that if the analyst had randomly sorted a weight $k = 6$ (in fact any number $5.1724 > k \geq 100$) then, the weight of reputation would have overrode the weight of the brokerage commission c, and thus his decision would have been to reveal a loyal recommendation r_L , leading to a loyal equilibrium. Before studying the reaction of k^* to parameters, it is important to precise that the fund manager will not necessarily follow the biased analyst.

3.2 When the fund manager can "de-bias" the recommendation

Since the volume V played by the fund manager depends heavily on his revised belief λ' it seems worthy to study when this revision is made in the opposite direction than the one indicated by the recommendation. The question is to know whether the revised belief will lead the fund manager to think that the high price x_H is more probable ($\lambda' > \lambda$), hence entailing a positive V, i.e. a purchase, or that it is less probable ($\lambda' < \lambda$), entailing a negative V, i.e. a sale.

Having in mind that we posited $\lambda = 1/2$, and with the following results (details in Annex A, p.25), we can compare λ' with λ :

$$\begin{aligned} \lambda'(r_H, y_H) &= 2z(1 - (1 - p)\pi) \\ \lambda'(r_L, y_L) &= 2(1 - z)(1 - p)\pi \\ \lambda'(r_H, y_L) &= 2(1 - z)[1 - (1 - p)\pi] \\ \lambda'(r_L, y_H) &= 2z(1 - p)\pi \end{aligned}$$

- Case 1. Whatever z, p and π under assumptions 1 and 2, it is simple to show that, when the two sources are corresponding, the revised belief always follows the direction indicated. There is an upward revision and purchase ($\lambda'(r_H, y_H) > \lambda$) when the analyst sends a favorable recommendation and the fund manager receives a high signal ; and there is an downward revision and sale ($\lambda'(r_L, y_L) < \lambda$) when both the recommendation and the fund manager's signal are unfavorable.

So the two interesting cases are when the recommendation conflicts with the fund manager's signal.

- Case 2. When (r_L, y_H), the analyst is pessimistic and the fund manager is optimistic. The fund manager will follow his own signal (and deviate from the analyst) if $\lambda'(r_L, y_H) > \lambda$, i.e. when:

$$z > \frac{1}{4(1 - p)\pi} \tag{27}$$

and will deviate from his signal and sell the stock (follow the analyst) in the contrary.

But, under assumption 2, sending r_L reveals that the analyst is loyal. As, under assumption 1, his signal is the most precise, actually the fund manager will always follow the analyst's recommendation (see proof in Appendix A)

- Case 3. When (r_H, y_L), the analyst is optimistic and the fund manager is pessimistic. The fund manager will follow his own signal and sell (and deviate from the analyst) if $\lambda'(r_H, y_L) < \lambda$, i.e. when

$$z > 1 - \frac{1}{4(1 - (1 - p)\pi)} \tag{28}$$

  and will deviate from his own signal and buy (and follow the analyst) in the contrary.

To go farther, consider that the right-hand side of equation (28) is a z-function depending on p. Then we can plot this function[8] to compare the values of z and p. In the zone above the z-curve, the fund manager will deviate from the analyst and sell; in the zone under the z-curve, the fund manager will follow the analyst and buy (see figure 4).

As we can see, the fund manager will deviate from the analyst when the precision of its own signal z is "not too inferior" to the analyst's signal precision p. That is what expresses condition (28). On the other hand, the more precise is the analyst, the more the fund manager has chances to follow him. To put it another way, the more p increases, the more z has chances to be under the curve, rather than above the curve.

This leads to the following proposition.

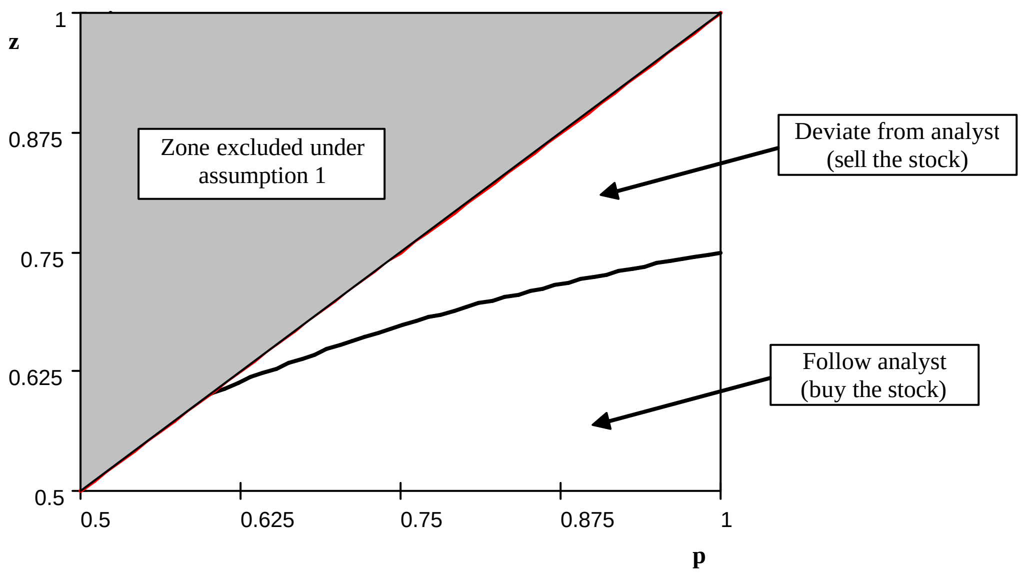
Proposition 3 When the fund manager's own signal and the analyst's recommendation are conflicting ($y \neq r$) :

(a) if the analyst is pessimistic (r_L, y_H), the fund manager will necessarily follow him, if the analyst is optimistic (r_H, y_L), the fund manager will follow him only in some cases

(b) When the analyst is optimistic, the fund manager will follow his own signal (and deviate from the analyst's recommendation) when its precision is "not too inferior" to the analyst's signal precision.

[8] As actually the probability π is a function of k^* which itself depends on our four-parameter set α, c, n and p, the function plotted here is much more complicated than its expression in (28) seems to be. However, it neither changes the results nor their interpretation. So to plot the graph we need to give some values to α, c, n and to express π as a function of p only. The graph here takes the same values as our previous numerical example, α = 20, c = 0.1 and n = 0.1. There always exists the two "zones" on the graph, whatever are the values of the parameters α, c, n.

Figure 4: Nature of the trade and signal's precision



This proposition joins up with anecdotal observations that fund manager can sometimes "de-bias" the analysts' recommendation (Boni & Womack 2002, p.30). Recall there always exist an ex-ante uncertainty about the intentions (loyalty/bias) of the analyst. The fund manager can never be sure that the analyst is biased. But the fact that the fund manager benefits from his private signal can sometimes help him to discard the analysts suspected of "hyping" their recommendations in order to generate trading commissions.

For example, following the very numerical example of section (3.1.3) except for z, the fund manager takes the opposite direction (he sells, because $\lambda'(r_H, y_L) < 0.5$) if $z > 0.692$. Through this "fund manager's signal", we must understand the importance of buy-side analysts, those members of the fund manager's team precisely in charge of giving an unbiased, though less precise, information.

3.3 The determinants of the probability of bias

Interpret (26) as a k-function of each parameters successively, for example as $k^*(c)$. Then it yields (see details in appendix B, p.28-30) :

$\frac{dk^*}{dc} > 0$	$\frac{dk^*}{d\alpha} > 0$	$\frac{dk^*}{dp} < 0$	$\frac{dk^*}{dn} < 0$

All else equal, the probability of bias ($1 - \pi = \frac{k^*}{100}$) increases with the percentage trading commission rate (c), and the size of the amount traded (α), but decreases with the analyst's signal precision (p) and with the size of the revision of analyst rating (n). These are the following propositions.

Proposition 4 The probability of bias increases with the commission rate.

This result shows that it is not inconsistent to think that the brokerage commission incentive has an important role in the recommendation bias of the analysts. It is congruent with studies showing a positive relation between the over-optimism of analysts and the situation of the brokerage …rm in terms of market shares or in terms of pro…t size (Aitken, Muthuswamy & Wong 2000, Irvine 2004).

Proposition 5 The probability of bias increases with the size of the trade.

This is similar to Hayes (1998) main result, according to witch the trading commission incentives strengthen when the size of the trade is high.

Now the next proposition is true under a slight condition (details in appendix B p.29), which is that if p takes a precise value near $1/2$ the probability can not be de…ned. Under the condition that p is di¤erent from this peculiar value, the following proposition holds.

Proposition 6 The probability of bias decreases with the precision of the analyst's signal

This is one of the main conclusions of herding models applied to …nance, where biased behaviors are held by the less precise, "dumb", low-ability, less experienced analysts (in particular Jackson, 2005). The proposition also illustrates tests showing a negative relation between analysts' forecast bias and several proxies for precision (analysts in the top ranking, or analysts with upward career paths (for example Hong & Kubik 2003).

Proposition 7 The probability of bias decreases with the size of the variation of analyst rating.

With analyst rating, the fund manager has a credible threat. The larger the variation of analyst rating, the more the analyst is deterred of biasing his recommendation. As the proposition 4, this last proposition is a new one, derived from the new way of taking into account the weight of reputation. It seems to support the claims for systematic and transparent analyst rating. For example, McNamee (2002) states that rating is "the only cure" to analysts' bias. Firms like Bureau van Dijk, Reuters or Bloomberg sell rating software which rate the analyst comparing the recommendation to realized evolution of prices or earnings. Many funds also build their in-house rating, called "broker review" by professionals. The ex-post surveillance of the quality of advice has a rationale.

4 Discussion

The limits of the sort of model presented concern both prices and volumes. First, the fund manager trade does not have any impact on prices –hence he cannot form any strategies conditional to this impact. Second, the manager cannot endogenously choose not to participate in the market (in the Hayes-Jackson model the may not participate according to an exogenous probability of short sale constraint).

However, it helps shed some light on the debates about the reform of "sell-side" brokerage houses analysts' payment mode. In recent reports (FSA 2003, AMF 2005b), Financial Market Regulators explore one way of limiting the trading commission incentives that biases recommendations. They implicitely admit that deontological codes exhorting analysts to be loyal do not su¢ce, by promoting the unbundling of brokerage commissions.

The idea is that the fund manager can allow a percentage of the trading commission to an analysts bureau that is di¤erent from the brokerage house executing the market order. For example, if the total commission is c £ V, he can allow say 50% of the total to the brokerage house A for payment of the execution of the trade, and the remaining 50% of the total to a brokerage house B where an analyst gave a satisfactory recommendation. But there are reasons to be skeptical about this reform. As it lowers the impact of the commission rate c on the incentive to bias, it takes in the right direction. But it is illusory to think that it will utterly suppress the incentive, as for the whole community of analysts still have an interest in generating trade. Furthermore there are many other practical reasons that drive fund managers and individual investors to pay less attention to unfavorable (and, hence, loyal) recommendations (see Boni & Womack 2002, p.110). The core of the problem is that the adviser is not independent : his payment depends on the nature of his advice.

So the most radical reform to ensure the independence of analysts would be to separate them from brokerage houses. If $c = 0$ the probability of bias is obviously zero. The 2002-2003 "General Settlement" by New York General Attorney provides for …nancing …nancial services independent from trading commissions.

Yet the e¢ciency of this reform this would probably be limited by a decrease in sell-side analysts' signal precision. They indeed bene…t from a daily access to the in-house traders and to company managers. This position gives them the informational advantage that justi…es our assumption 1. Independent analysts would certainly be separated from this informationally advantageous position.

Still the model shows two factors that may preserve high-precision sell-side analysts within brokerage houses, and though that may limit the trading commission incentives : 1) systematically develop a¤ordable analyst rating, and 2) develop "buy-side" analysis besides the fund managers. Whether developing these two factors is really a better solution for the financial community as a whole should be more explored.

Appendix

Appendix A. Revised beliefs

$\Pr(x \mid y, r)$ is defined as $\lambda'(y, r)$. Then,

$$\Pr(x \mid y, r) = \Pr(x \mid r)\frac{\Pr(y \mid x)}{\Pr(y)}$$

$\Pr(y \mid x)$ are given as the fund manager's signal. Other probabilities must be computed.

Computation of $\Pr(y)$

Use the partition of y over x:

$$\begin{aligned} \Pr(y_H) &= \Pr(y_H \mid x_H)\Pr(x_H) + \Pr(y_H \mid x_L)\Pr(x_L) \\ &= 0.5z + 0.5(1 - z) = 0.5 \end{aligned}$$

As the sum of the information set is 1, we have:

$$\Pr(y_L) = 0.5$$

It is unsurprising since $\Pr(x_H) = \lambda = 0.5 = \Pr(x_L)$.

Computation of $\Pr(x \mid r)$

From (1.4), examine every situation starting from r_H that can lead to x_H. I develop this first case and give only the result for the other cases.

Given the decision rule of the analyst, he can send r_H only if he received s_H in the case he's loyal, and whatever the signal in the case he's biased. Whether he's biased (probability $1 - \pi$) or loyal (π) do not depend on y as we will see, although the signal received depends on x. Writing extensively all the path leading to r_H starting from x_H, it yields:

$$\begin{aligned} \Pr(x_H \mid r_H) = {} & \Pr(y_H \mid x_H)[\Pr(x_H \mid s_H)\{\Pr(loyal) + \Pr(biais¶)\} + \Pr(x_H \mid s_L)\Pr(biais¶)] \\ & + \Pr(y_L \mid x_H)[\Pr(x_H \mid s_H)\{\Pr(loyal) + \Pr(biais¶)\} + \Pr(x_H \mid s_L)\Pr(biais¶)] \end{aligned}$$

$$= z[p\{\pi + 1 - \pi\} + (1 - p)(1 - \pi)] + (1 - z)[p\{\pi + (1 - \pi)\} + (1 - p)(1 - \pi)]$$

Because of the partition on y and this can be simplified:

$$\begin{aligned}
\Pr(x_H \mid r_H) &= \underbrace{\Pr(x_H \mid s_H)}_{\text{if } s_H \text{ then } r_H \text{ whatever the type}} + \underbrace{\Pr(x_H \mid s_L)\Pr(biaised)}_{\text{if } s_L \text{ then } r_H \text{ if biased analyst}} \\
&= p + (1 - p)(1 - \pi) \\
&= 1 - (1 - p)\pi
\end{aligned}$$

Using the same method for the other cases, it gives: :

$$\begin{aligned}
\Pr(x_H \mid r_H) &= 1 - (1 - p)\pi && (29) \\
\Pr(x_H \mid r_L) &= (1 - p)\pi \\
\Pr(x_L \mid r_H) &= p\pi \\
\Pr(x_L \mid r_L) &= 1 - p\pi
\end{aligned}$$

Now compute the revised beliefs.

Revised belief $\lambda'(y_H, r_H)$

Remember that $\lambda'(y_H, r_H) = \Pr(x_H \mid y_H, r_H)$. Then,

$$\begin{aligned}
\lambda'(y_H, r_H) &= \Pr(x_H \mid r_H)\frac{\Pr(y_H \mid x_H)}{\Pr(y_H)} && (30) \\
&= 1 - (1 - p)\pi\frac{z}{(1/2)} \\
&= 2z(1 - (1 - p)\pi)
\end{aligned}$$

Revised belief $\lambda'(y_L, r_L)$

Idem,

$$\begin{aligned}
\lambda'(y_L, r_L) &= \Pr(x_H \mid r_L)\frac{\Pr(y_L \mid x_H)}{\Pr(y_L)} && (31) \\
&= 2(1 - z)(1 - p)\pi
\end{aligned}$$

Revised belief $\lambda'(y_L, r_H)$

idem,

$$\begin{aligned}
\lambda'(y_L, r_H) &= \Pr(x_H \mid r_H)\frac{\Pr(y_L \mid x_H)}{\Pr(y_L)} \\
&= 2(1 - z)[1 - (1 - p)\pi]
\end{aligned} \tag{32}$$

Revised belief $\lambda'(y_H, r_L)$

Idem,

$$\begin{aligned}
\lambda'(y_H, r_L) &= \Pr(x_H \mid r_L)\frac{\Pr(y_H \mid x_H)}{\Pr(y_H)} \\
&= 2z(1 - p)\pi
\end{aligned} \tag{33}$$

Each of the four revised belief mentioned has the revised belief about x_L as a complementary event ; i.e. $\Pr(x_L \mid y, r) = 1 - \lambda'(y, r)$. Hence summing the λ' has no particular meaning.

Proof that the fund manager follows the analyst when r_L

The aim is to show that condition (27) is not compatible with assumption 1. Write simultaneously condition (27) and assumption 1:

$$p > z > \frac{1}{4(1 - p)\pi} \tag{34}$$

$$\iff p(1 - p) > \frac{1}{4\pi} \tag{35}$$

On the right-hand side of equation (35), the minimum value of $1/4\pi$ is $1/4$. On the left-hand side of the equation, recall that the minimum value of p tends towards $1/2$ but is superior to $1/2$ since there always exists a z between the two. Hence the maximum of $p(1 - p)$ tends towards $1/4$ but is inferior to $1/4$, so that equation (35) is never true.

Appendix B. Computation of k*

Given the uniform probability density function chosen for π, replace π by $1 - \frac{k^*}{100}$ in the expression above, then isolate k^*:

$$k^* = \frac{\frac{c\alpha}{2}(1 - 2(1-p)\pi))}{2n(2p-1)}$$

which yields

$$\begin{aligned}
k^* &= \frac{\frac{c\alpha}{2}(1 - 2(1-p)(1 - \frac{k^*}{100}))}{2n(2p-1)} \\
\iff \quad k^* &= \frac{\frac{c\alpha}{2} - \frac{c\alpha}{2}(1 - \frac{k^*}{100})(2(1-p))}{2n(2p-1)} \\
\iff \quad k^* + \frac{\frac{c\alpha}{2}(1 - \frac{k^*}{100})(2(1-p))}{2n(2p-1)} &= \frac{\frac{c\alpha}{2}}{2n(2p-1)} \\
\iff \quad k^* \left(1 - \frac{\frac{c\alpha}{2}(2(1-p))}{100 \times 2n(2p-1)}\right) &= \frac{\frac{c\alpha}{2}}{2n(2p-1)} - \frac{\frac{c\alpha}{2}(2(1-p)}{2n(2p-1)} \\
\iff \quad k^* &= \left(\frac{\frac{c\alpha}{2} - \frac{c\alpha}{2}(2(1-p))}{2n(2p-1)}\right)\left(\frac{100 \times 2n(2p-1)}{100 \times 2n(2p-1) - \frac{c\alpha}{2}2(1-p)}\right) \\
\iff \quad k^* &= \frac{100\frac{c\alpha}{2}(1 - 2(1-p))}{100 \times 2n(2p-1) - c\alpha(1-p)} \\
\iff \quad k^* &= \frac{c\alpha(1 - 2(1-p))}{4n(2p-1) - \frac{2}{10^2}c\alpha(1-p)}
\end{aligned}$$

Note that the upper bound parameter of the uniform probability function (10^2) is clearly identified ; chosen greater parameters(e.g. 10^9) in order to get "closer" to the initial support of $0 < k < 1$ do not changes the result but simply blurs their interpretations.

Before interpreting the sign of the derivatives, note the sign of the following expressions:

$$\begin{aligned}
1 &\le [1 - 2(p-1)] < 2 \\
0 &< [2p-1] \le 1
\end{aligned}$$

Derivative w.r.t. c

$\frac{dk^*}{dc}$ is of the sign of:

$$\begin{aligned}
&\alpha(1 - 2(1-p))\left(4n(2p-1) - \frac{2c\alpha}{10^2}(1-p)\right) + c\alpha(1 - 2(1-p)\frac{2\alpha}{10^2}(1-p) \\
= \; &\alpha(1 - 2(1-p)4n(2p-1) > 0
\end{aligned}$$

k^* is not defined when:

$$4n(2p-1)-\frac{2}{10^2}c\alpha(1-p)=0$$
$$\iff \quad c=\frac{200n(2p-1)}{\alpha(1-p)}$$

which is generally higher than one (remember that $c\in[0,1]$). For example with $\{p=0.8,\ n=0.1,\ \alpha=20\}$, c must be different than 3.

Derivative w.r.t. α

$\frac{dk^*}{d\alpha}$ is of the sign of:

$$\begin{aligned}
& c(1-2(1-p))\left(4n(2p-1)-\frac{2c\alpha}{10^2}(1-p)\right)+c\alpha(1-2(1-p))\frac{2c}{10^2}(1-p) \\
= \; & c(1-2(1-p)4n(2p-1)>0
\end{aligned}$$

The derivative is positive except on a vertical asymptote at the point:

$$\alpha=\frac{200n(2p-1)}{c(1-p)}$$

For example if $\{p=0.8,\ n=0.1,\ c=0.1\}$ $\alpha=600$.

Derivative w.r.t. p

$\frac{dk^*}{dp}$ is of the sign of:

$$\begin{aligned}
& 2c\alpha(4n(2p-1))-c\alpha(1-2(1-p)(8n+\frac{2c\alpha}{10^2}) \\
= \; & \frac{2\alpha^2c^2}{10^2}(2p+1)(p-1)<0
\end{aligned}$$

The derivative is negative. Except for the vertical asymptote at point:

$$p=\frac{4n+\frac{c\alpha}{50}}{8n+\frac{c\alpha}{50}}$$

The asymptote is always near $1/2$, and k^* is negative between $1/2$ and the asymptote. It means that for very low precision of his signal, the analyst is systematically loyal, then after the asymptote the probability is very high and decreases slowly. For example with $\{\alpha=20,\ n=0.1,\ c=0.1\}$ the asymptote is $p=0.523$, and the interpretation holds for $p>0.523$. As $p<0.523$ is a very poor precision, and since $0.5<z<p$ we can imagine that the funds managers's precision is relatively close from the one of the analyst, hence a biased recommendation has no chances to succeed. In the contrary, if precision is just a bit superior to the asymptote, the expectation to generate trading commissions offsets the expected loss in reputation.

Derivative w.r.t. n

$\frac{dk^*}{dn}$ is of the sign of

$$-\alpha c(1-2(1-p)4(2p-1) < 0$$

and k^* is not defined for

$$n = \frac{c\alpha(1-p)}{200(2p-1)}$$

which is quite near to zero for large ranges of parameters (except when p tends towards $1/2$, then n is infinite if c tends towards 1). For example in our base case $\{p = 0.8, c = 0.1, \alpha = 20\}$ the asymptote is at point $n = 0.0006$.

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